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Chapter 1

Classes

1.1 quad – Imaginary Quadratic Field

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1.1.1 ReducedQuadraticForm – Reduced Quadratic Form Class

Initialize (Constructor)

ReducedQuadraticForm(`f`**:** *list*, `unit`**:** *list*) → *ReducedQuadraticForm*

Create ReducedQuadraticForm object.

`f`, `unit` must be list of 3 integers `[a, b, c]`, representing a quadratic form $ax^2 + bxy + cy^2$. `unit` represents the unit form.

Operations

operator	explanation
`M * N`	Return the composition form of `M` and `N`.
`M ** a`	Return the a-th powering of `M`.
`M / N`	Division of form.
`M == N`	Return whether `M` and `N` are equal or not.
`M != N`	Return whether `M` and `N` are unequal or not.

Methods

1.1.1.1 inverse

inverse(self) → ***ReducedQuadraticForm***

Return the inverse of self.

1.1.1.2 disc

disc(self) → ***ReducedQuadraticForm***

Return the discriminant of self.

1.1.2 ClassGroup – Class Group Class

Initialize (Constructor)

ClassGroup(disc: ***integer*****, cl:** ***integer*****, element:** ***integer*****=None)**
→ ***ClassGroup***

Create ClassGroup object.

Methods

1.1.3 class_formula

class_formula(d: *integer*, uprbd: *integer*) → *integer*

Return the approximation of class number h with discriminant `d` using class formula.

$$\text{class formula } h = \frac{\sqrt{|\mathtt{d}|}}{\pi} \prod_p \left(1 - \left(\frac{\mathtt{d}}{p}\right) \frac{1}{p}\right)^{-1}.$$

Input number `d` must be int, long or **Integer**.

1.1.4 class_number

class_number(d: *integer*, limit_of_d: *integer*=1000000000) → *integer*

Return the class number with the discriminant `d` by counting reduced forms.

`d` is not only fundamental discriminant.

Input number `d` must be int, long or **Integer**.

1.1.5 class_group

class_group(d: *integer*, limit_of_d: *integer*=1000000000) → *integer*

Return the class number and the class group with the discriminant `d` by counting reduced forms.

`d` is not only fundamental discriminant.

Input number `d` must be int, long or **Integer**.

1.1.6 class_number_bsgs

class_number_bsgs(d: *integer*) → *integer*

Return the class number with the discriminant `d` using Baby-step Giant-step algorithm.

`d` is not only fundamental discriminant.

Input number `d` must be int, long or **Integer**.

1.1.7 class_group_bsgs

class_group_bsgs(d: ***integer*****,** **cl:** ***integer*****,** **qin:** ***list*****)**
→ ***integer***

Return the construction of the class group of order p^{exp} with the discriminant disc, where `q`$in = [p, exp]$.

Input number `d`, `cl` must be int, long or **Integer**.

Examples

```
>>> quad.class_formula(-1200, 100000)
12
>>> quad.class_number(-1200)
12
>>> quad.class_group(-1200)
(12, [ReducedQuadraticForm(1, 0, 300), ReducedQuadraticForm(3, 0, 100),
ReducedQuadraticForm(4, 0, 75), ReducedQuadraticForm(12, 0, 25),
ReducedQuadraticForm(7, 2, 43), ReducedQuadraticForm(7, -2, 43),
ReducedQuadraticForm(16, 4, 19), ReducedQuadraticForm(16, -4, 19),
ReducedQuadraticForm(13, 10, 25), ReducedQuadraticForm(13, -10, 25),
ReducedQuadraticForm(16, 12, 21), ReducedQuadraticForm(16, -12, 21)])
>>> quad.class_number_bsgs(-1200)
12L
>>> quad.class_group_bsgs(-1200, 12, [3, 1])
([ReducedQuadraticForm(16, -12, 21)], [[3L]])
>>> quad.class_group_bsgs(-1200, 12, [2, 2])
([ReducedQuadraticForm(12, 0, 25), ReducedQuadraticForm(4, 0, 75)],
[[2L], [2L, 0]])
```

Bibliography