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Chapter 1

Classes

1.1 poly.termorder – term orders

- **Classes**
 - † **TermOrderInterface**
 - † **UnivarTermOrder**
 - **MultivarTermOrder**
- **Functions**
 - **weight_order**

1.1.1 TermOrderInterface – interface of term order

Initialize (Constructor)

TermOrderInterface(comparator: *function*) → *TermOrderInterface*

A term order is primarily a function, which determines precedence between two terms (or monomials). By the precedence, all terms are ordered.

More precisely in terms of `Python`, a term order accepts two tuples of integers, each of which represents power indices of the term, and returns 0, 1 or -1 just like `cmp` built-in function.

A `TermOrder` object provides not only the precedence function, but also a method to format a string for a polynomial, to tell degree, leading coefficients, etc.

comparator accepts two tuple-like objects of integers, each of which represents power indices of the term, and returns 0, 1 or -1 just like `cmp` built-in function.

This class is abstract and should not be instantiated. The methods below have to be overridden.

Methods

1.1.1.1 `cmp`

`cmp(self, left: tuple, right: tuple) → integer`

Compare two index tuples left and right and determine precedence.

1.1.1.2 `format`

`format(self, polynom: polynomial, **keywords: dict)`
→ *string*

Return the formatted string of the polynomial polynom.

1.1.1.3 `leading_coefficient`

`leading_coefficient(self, polynom: polynomial) → CommutativeRingElement`

Return the leading coefficient of polynomial polynom with respect to the term order.

1.1.1.4 `leading_term`

`leading_term(self, polynom: polynomial) → tuple`

Return the leading term of polynomial polynom as tuple of (degree index, coefficient) with respect to the term order.

1.1.2 `UnivarTermOrder` – term order for univariate polynomials

Initialize (Constructor)

`UnivarTermOrder(comparator: function) → UnivarTermOrder`

There is one unique term order for univariate polynomials. It's known as degree.

One thing special to univariate case is that powers are not tuples but bare integers. According to the fact, method signatures also need be translated from the definitions in `TermOrderInterface`, but its easy, and we omit some explanations.

`comparator` can be any callable that accepts two integers and returns 0, 1 or -1 just like `cmp`, i.e. if they are equal it returns 0, first one is greater 1, and otherwise -1. Theoretically acceptable comparator is only the `cmp` function.

This class inherits **TermOrderInterface**.

Methods

1.1.2.1 format

```
format(self, polynom: polynomial, varname: string='X', reverse:  
bool=False)  
    → string
```

Return the formatted string of the polynomial polynom.

- polynom must be a univariate polynomial.
- varname can be set to the name of the variable.
- reverse can be either `True` or `False`. If it's `True`, terms appear in reverse (descending) order.

1.1.2.2 degree

```
degree(self, polynom: polynomial) → integer
```

Return the degree of the polynomial polynom.

1.1.2.3 tail_degree

```
tail_degree(self, polynom: polynomial) → integer
```

Return the least degree among all terms of the polynom.

This method is *experimental*.

1.1.3 MultivarTermOrder – term order for multivariate polynomials

Initialize (Constructor)

```
MultivarTermOrder(comparator: function) → MultivarTermOrder
```

This class inherits **TermOrderInterface**.

Methods

1.1.3.1 format

```
format(self, polynom: polynomial, varname: tuple=None, reverse:
bool=False, **kwds: dict)
    → string
```

Return the formatted string of the polynomial polynom.

An additional argument varnames is required to name variables.

- polynom is a multivariate polynomial.
- varnames is the sequence of the variable names.
- reverse can be either True or False. If it's True, terms appear in reverse (descending) order.

1.1.4 weight_order – weight order

```
weight_order(weight: sequence, tie_breaker: function=None)
    → function
```

Return a comparator of weight ordering by weight.

Let w denote the weight. The weight ordering is defined for arguments x and y that $x < y$ if $w \cdot x < w \cdot y$ or $w \cdot x == w \cdot y$ and tie breaker tells $x < y$.

The option tie_breaker is another comparator that will be used if dot products with the weight vector leaves arguments tie. If the option is None (default) and a tie breaker is indeed necessary to order given arguments, a TypeError is raised.

Examples

```
>>> w = termorder.MultivarTermOrder(
...     termorder.weight_order((6, 3, 1), cmp))
>>> w.cmp((1, 0, 0), (0, 1, 2))
1
```

Bibliography