Contents

Chapter 1

Classes

1.1 permute – permutation (symmetric) group

- **Classes**
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1.1.1 Permute – element of permutation group

Initialize (Constructor)

Permute(`value`: *list/tuple*, `key`: *list/tuple*) → Permute

Permute(`val_key`: *dict*) → Permute

Permute(`value`: *list/tuple*, `key`: *int*=None) → Permute

Create an element of a permutation group.

An instance will be generated with "normal" way. That is, we input a `key`, which is a list of (indexed) all elements from some set, and a `value`, which is a list of all permuted elements.

Normally, you input two lists (or tuples) `value` and `key` with same length. Or you can input `val_key` as a dict whose `values()` is a list "value" and `keys()` is a list "key" in the sense of above. Also, there are some short-cut for inputting `key`:

- If key is $[1,\ 2, \ldots, N]$, you do not have to input `key`.
- If key is $[0,\ 1, \ldots, N-1]$, input 0 as `key`.
- If key equals the list arranged through `value` in ascending order, input 1.
- If key equals the list arranged through `value` in descending order, input -1.

Attributes

key :
It expresses `key`.

data :
†It expresses indexed form of `value`.

Operations

operator	explanation
`A==B`	Check equality for A's value and B's one and A's key and B's one.
`A*B`	right multiplication (that is, $A \circ B$ with normal mapping way)
`A/B`	division (that is, $A \circ B^{-1}$)
`A**B`	powering
`A.inverse()`	inverse
`A[c]`	the element of `value` corresponding to `c` in `key`
`A(lst)`	permute `lst` with A

Examples

```
>>> p1 = permute.Permute(['b','c','d','a','e'], ['a','b','c','d','e'])
>>> print p1
['a', 'b', 'c', 'd', 'e'] -> ['b', 'c', 'd', 'a', 'e']
>>> p2 = permute.Permute([2, 3, 0, 1, 4], 0)
>>> print p2
[0, 1, 2, 3, 4] -> [2, 3, 0, 1, 4]
>>> p3 = permute.Permute(['c','a','b','e','d'], 1)
>>> print p3
['a', 'b', 'c', 'd', 'e'] -> ['c', 'a', 'b', 'e', 'd']
>>> print p1 * p3
['a', 'b', 'c', 'd', 'e'] -> ['d', 'b', 'c', 'e', 'a']
>>> print p3 * p1
['a', 'b', 'c', 'd', 'e'] -> ['a', 'b', 'e', 'c', 'd']
>>> print p1 ** 4
['a', 'b', 'c', 'd', 'e'] -> ['a', 'b', 'c', 'd', 'e']
>>> p1['d']
'a'
>>> p2([0, 1, 2, 3, 4])
[2, 3, 0, 1, 4]
```

Methods

1.1.1.1 setKey – change key

setKey(self, key: *list/tuple*) → *Permute*

Set other key.

key must be list or tuple with same length to **key**.

1.1.1.2 getValue – get "value"

getValue(self) → *list*

Return (not data) value of self.

1.1.1.3 getGroup – get PermGroup

getGroup(self) → *PermGroup*

Return **PermGroup** to which self belongs.

1.1.1.4 numbering – give the index

numbering(self) → *int*

Number self in the permutation group. (Slow method)

The numbering is made to fit the following inductive definition for dimension of the permutation group.
If numbering of $[\sigma_1,\ \sigma_2, ..., \sigma_{n-2},\ \sigma_{n-1}]$ on $(n-1)$-dimension is k, numbering of $[\sigma_1,\ \sigma_2, ..., \sigma_{n-2}, \sigma_{n-1}, n]$ on n-dimension is k and numbering of $[\sigma_1,\ \sigma_2, ..., \sigma_{n-2},\ n,\ \sigma_{n-1}]$ on n-dimension is $k + (n-1)!$, and so on. (See Room of Points And Lines, part 2, section 15, paragraph 2 (Japanese))

1.1.1.5 order – order of the element

order(self) → *int/long*

Return order as the element of group.

This method is faster than general group method.

1.1.1.6 ToTranspose – represent as transpositions

ToTranspose(self) → *ExPermute*

Represent self as a composition of transpositions.

Return the element of **ExPermute** with transpose (2-dimensional cyclic) type. It is recursive program, and it would take more time than the method **ToCyclic**.

1.1.1.7 ToCyclic – corresponding ExPermute element

ToCyclic(self) → *ExPermute*

Represent self as a composition of cyclic representations.

Return the element of **ExPermute**. †This method decomposes self into coprime cyclic permutations, so each cyclic is commutative.

1.1.1.8 sgn – sign of the permutation

sgn(self) → *int*

Return the sign of permutation group element.

If self is even permutation, that is, self can be written as a composition of an even number of transpositions, it returns 1. Otherwise,that is, for odd permutation, it returns -1.

1.1.1.9 types – type of cyclic representation

types(self) → *list*

Return cyclic type defined by each cyclic permutation element length.

1.1.1.10 ToMatrix – permutation matrix

ToMatrix(self) → Matrix

Return permutation matrix.

The row and column correspond to `key`. If `self` G satisfies $G[a] = b$, then $(a,\ b)$-element of the matrix is 1. Otherwise, the element is 0.

Examples

```
>>> p = Permute([2,3,1,5,4])
>>> p.numbering()
28
>>> p.order()
6
>>> p.ToTranspose()
[(4,5)(1,3)(1,2)](5)
>>> p.sgn()
-1
>>> p.ToCyclic()
[(1,2,3)(4,5)](5)
>>> p.types()
'(2,3)type'
>>> print p.ToMatrix()
0 1 0 0 0
0 0 1 0 0
1 0 0 0 0
0 0 0 0 1
0 0 0 1 0
```

1.1.2 ExPermute – element of permutation group as cyclic representation

Initialize (Constructor)

ExPermute(`dim`: *int*, `value`: *list*, `key`: *list*=None) → ExPermute

Create an element of a permutation group.

An instance will be generated with "cyclic" way. That is, we input a `value`, which is a list of tuples and each tuple expresses a cyclic permutation. For example, $(\sigma_1,\ \sigma_2,\ \sigma_3, \ldots, \sigma_k)$ is one-to-one mapping, $\sigma_1 \mapsto \sigma_2,\ \sigma_2 \mapsto \sigma_3, \ldots, \sigma_k \mapsto \sigma_1$.

`dim` must be positive integer, that is, an instance of int, long or **Integer**. `key` should be a list whose length equals `dim`. Input a list of tuples whose elements are in `key` as `value`. Note that you can abbreviate `key` if `key` has the form $[1,\ 2, \ldots, N]$. Also, you can input 0 as `key` if `key` has the form $[0,\ 1, \ldots, N-1]$.

Attributes

dim :
It expresses `dim`.

key :
It expresses `key`.

data :
†It expresses indexed form of `value`.

Operations

operator	explanation
`A==B`	Check equality for A's value and B's one and A's key and B's one.
`A*B`	right multiplication (that is, $A \circ B$ with normal mapping way)
`A/B`	division (that is, $A \circ B^{-1}$)
`A**B`	powering
`A.inverse()`	inverse
`A[c]`	the element of `value` corresponding to `c` in `key`
`A(lst)`	permute `lst` with A
`str(A)`	simple representation. use **simplify**.
`repr(A)`	representation

Examples

```
>>> p1 = permute.ExPermute(5, [('a', 'b')], ['a','b','c','d','e'])
>>> print p1
[('a', 'b')] <['a', 'b', 'c', 'd', 'e']>
>>> p2 = permute.ExPermute(5, [(0, 2), (3, 4, 1)], 0)
>>> print p2
[(0, 2), (1, 3, 4)] <[0, 1, 2, 3, 4]>
>>> p3 = permute.ExPermute(5,[('b','c')],['a','b','c','d','e'])
>>> print p1 * p3
[('a', 'b'), ('b', 'c')] <['a', 'b', 'c', 'd', 'e']>
>>> print p3 * p1
[('b', 'c'), ('a', 'b')] <['a', 'b', 'c', 'd', 'e']>
>>> p1['c']
'c'
>>> p2([0, 1, 2, 3, 4])
[2, 4, 0, 1, 3]
```

Methods

1.1.2.1 setKey – change key

setKey(self, key: *list*) → *ExPermute*

Set other key.

key must be a list whose length equals **dim**.

1.1.2.2 getValue – get "value"

getValue(self) → *list*

Return (not data) value of self.

1.1.2.3 getGroup – get PermGroup

getGroup(self) → *PermGroup*

Return **PermGroup** to which self belongs.

1.1.2.4 order – order of the element

order(self) → *int/long*

Return order as the element of group.

This method is faster than general group method.

1.1.2.5 ToNormal – represent as normal style

ToNormal(self) → *Permute*

Represent self as an instance of **Permute**.

1.1.2.6 simplify – use simple value

simplify(self) → *ExPermute*

Return the more simple cyclic element.

†This method uses **ToNormal** and **ToCyclic**.

1.1.2.7 sgn – sign of the permutation

sgn(self) → *int*

Return the sign of permutation group element.

If self is even permutation, that is, self can be written as a composition of an even number of transpositions, it returns 1. Otherwise,that is, for odd permutation, it returns −1.

Examples

```
>>> p = permute.ExPermute(5, [(1, 2, 3), (4, 5)])
>>> p.order()
6
>>> print p.ToNormal()
[1, 2, 3, 4, 5] -> [2, 3, 1, 5, 4]
>>> p * p
[(1, 2, 3), (4, 5), (1, 2, 3), (4, 5)] <[1, 2, 3, 4, 5]>
>>> (p * p).simplify()
[(1, 3, 2)] <[1, 2, 3, 4, 5]>
```

1.1.3 PermGroup – permutation group

Initialize (Constructor)

PermGroup(key: *int/long*) → PermGroup

PermGroup(key: *list/tuple*) → PermGroup

Create a permutation group.

Normally, input list as `key`. If you input some integer N, `key` is set as $[1,\ 2, \ldots, N]$.

Attributes

key :
It expresses `key`.

Operations

operator	explanation
`A==B`	Check equality for A's value and B's one and A's key and B's one.
`card(A)`	same as **grouporder**
`str(A)`	simple representation
`repr(A)`	representation

Examples

```
>>> p1 = permute.PermGroup(['a','b','c','d','e'])
>>> print p1
['a','b','c','d','e']
>>> card(p1)
120L
```

Methods

1.1.3.1 createElement – create an element from seed

createElement(self, seed: *list/tuple/dict*) → *Permute*

createElement(self, seed: *list*) → *ExPermute*

Create new element in self.

seed must be the form of "value" on **Permute** or **ExPermute**

1.1.3.2 identity – group identity

identity(self) → *Permute*

Return the identity of self as normal type.

For cyclic type, use **identity_c**.

1.1.3.3 identity_c – group identity as cyclic type

identity_c(self) → *ExPermute*

Return permutation group identity as cyclic type.

For normal type, use **identity**.

1.1.3.4 grouporder – order as group

grouporder(self) → *int/long*

Compute the order of self as group.

1.1.3.5 randElement – random permute element

randElement(self) → *Permute*

Create random new element as normal type in self.

Examples

```
>>> p = permute.PermGroup(5)
>>> print p.createElement([3, 4, 5, 1, 2])
[1, 2, 3, 4, 5] -> [3, 4, 5, 1, 2]
>>> print p.createElement([(1, 2), (3, 4)])
[(1, 2), (3, 4)] <[1, 2, 3, 4, 5]>
>>> print p.identity()
[1, 2, 3, 4, 5] -> [1, 2, 3, 4, 5]
>>> print p.identity_c()
[] <[1, 2, 3, 4, 5]>
>>> p.grouporder()
120L
>>> print p.randElement()
[1, 2, 3, 4, 5] -> [3, 4, 5, 2, 1]
```

Bibliography