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Chapter 1

Classes

1.1 matrix – matrices

- **Classes**
 - **Matrix**
 - **SquareMatrix**
 - **RingMatrix**
 - **RingSquareMatrix**
 - **FieldMatrix**
 - **FieldSquareMatrix**
 - **MatrixRing**
 - **Subspace**
- **Functions**
 - **createMatrix**
 - **identityMatrix**
 - **unitMatrix**
 - **zeroMatrix**

The module `matrix` has also some exception classes.

MatrixSizeError : Report contradicting given input to the matrix size.

VectorsNotIndependent : Report column vectors are not independent.

NoInverseImage : Report any inverse image does not exist.

NoInverse : Report the matrix is not invertible.

This module using following type:

compo : compo must be one of these forms below.

- concatenated row lists, such as $[1,2]+[3,4]+[5,6]$.
- list of row lists, such as $[[1,2], [3,4], [5,6]]$.
- list of column tuples, such as $[(1, 3, 5), (2, 4, 6)]$.
- list of vectors whose dimension equals column, such as `vector.Vector([1, 3, 5])`, `vector.Vector([2, 4, 6])`.

The examples above represent the same matrix form as follows:

$$\begin{array}{cc} 1 & 2 \\ 3 & 4 \\ 5 & 6 \end{array}$$

1.1.1 Matrix – matrices

Initialize (Constructor)

```
Matrix(row: integer, column: integer, compo: compo=0, coeff_ring:
CommutativeRing=0)
→ Matrix
```

Create new matrices object.

†This constructor automatically changes the class to one of the following class: **RingMatrix**, **RingSquareMatrix**, **FieldMatrix**, **FieldSquareMatrix**.

Your input determines the class automatically by examining the matrix size and the coefficient ring. **row** and **column** must be integer, and **coeff_ring** must be an instance of **Ring**. Refer to **compo** for information about **compo**. If you abbreviate **compo**, it will be deemed to all zero list.

The list of expected inputs and outputs is as following:

- Matrix(**row**, **column**, **compo**, **coeff_ring**)
→ the **row**×**column** matrix whose elements are **compo** and coefficient ring is **coeff_ring**
- Matrix(**row**, **column**, **compo**)
→ the **row**×**column** matrix whose elements are **compo** (The coefficient ring is automatically determined.)
- Matrix(**row**, **column**, **coeff_ring**)
→ the **row**×**column** matrix whose coefficient ring is **coeff_ring** (All elements are 0 in **coeff_ring**.)
- Matrix(**row**, **column**)
→ the **row**×**column** matrix (The coefficient matrix is **Integer**. All elements are 0.)

Attributes

row : The row size of the matrix.

column : The column size of the matrix.

coeff_ring : The coefficient ring of the matrix.

compo : The elements of the matrix.

Operations

operator	explanation
<code>M==N</code>	Return whether M and N are equal or not.
<code>M[i, j]</code>	Return the coefficient of i-th row, j-th column term of matrix M.
<code>M[i]</code>	Return the vector of i-th column term of matrix M.
<code>M[i, j]=c</code>	Replace the coefficient of i-th row, j-th column term of matrix M by c.
<code>M[j]=c</code>	Replace the vector of i-th column term of matrix M by vector c.
<code>c in M</code>	Check whether some element of M equals c.
<code>repr(M)</code>	Return the repr string of the matrix M. string represents list concatenated row vector lists.
<code>str(M)</code>	Return the str string of the matrix M.

Examples

```
>>> A = matrix.Matrix(2, 3, [1,0,0]+[0,0,0])
>>> A.__class__.__name__
'RingMatrix'
>>> B = matrix.Matrix(2, 3, [1,0,0,0,0,0])
>>> A == B
True
>>> B[1, 1] = 0
>>> A != B
True
>>> B == 0
True
>>> A[1, 1]
1
>>> print repr(A)
[1, 0, 0]+[0, 0, 0]
>>> print str(A)
1 0 0
0 0 0
```

Methods

1.1.1.1 map – apply function to elements

`map(self, function: function) → Matrix`

Return the matrix whose elements is applied function to.

†The function map is an analogy of built-in function `map`.

1.1.1.2 reduce – reduce elements iteratively

`reduce(self, function: function, initializer: RingElement=None)
→ RingElement`

Apply function from upper-left to lower-right, so as to reduce the iterable to a single value.

†The function map is an analogy of built-in function `reduce`.

1.1.1.3 copy – create a copy

`copy(self) → Matrix`

create a copy of `self`.

†The matrix generated by the function is same matrix to `self`, but not same as a instance.

1.1.1.4 set – set compo

`set(self, compo: compo) → (None)`

Substitute the list `compo` for `compo`.

`compo` must be the form of `compo`.

1.1.1.5 setRow – set m-th row vector

`setRow(self, m: integer, arg: list/Vector) → (None)`

Substitute the list/Vector arg as m-th row.

The length of arg must be same to self.column.

1.1.1.6 setColumn – set n-th column vector

`setColumn(self, n: integer, arg: list/Vector) → (None)`

Substitute the list/Vector arg as n-th column.

The length of arg must be same to self.row.

1.1.1.7 getRow – get i-th row vector

`getRow(self, i: integer) → Vector`

Return i-th row in form of self.

The function returns a row vector (an instance of **Vector**).

1.1.1.8 getColumn – get j-th column vector

`getColumn(self, j: integer) → Vector`

Return j-th column in form of self.

1.1.1.9 swapRow – swap two row vectors

`swapRow(self, m1: integer, m2: integer) → (None)`

Swap self's m1-th row vector and m2-th row one.

1.1.1.10 swapColumn – swap two column vectors

```
swapColumn(self, n1: integer, n2: integer) → (None)
```

Swap self's n1-th column vector and n2-th column one.

1.1.1.11 insertRow – insert row vectors

```
insertRow(self, i: integer, arg: list/Vector/Matrix)  
→ (None)
```

Insert row vectors arg to i-th row.

arg must be list, **Vector** or **Matrix**. The length (or **column**) of it should be same to the column of self.

1.1.1.12 insertColumn – insert column vectors

```
insertColumn(self, j: integer, arg: list/Vector/Matrix)  
→ (None)
```

Insert column vectors arg to j-th column.

arg must be list, **Vector** or **Matrix**. The length (or **row**) of it should be same to the row of self.

1.1.1.13 extendRow – extend row vectors

```
extendRow(self, arg: list/Vector/Matrix) → (None)
```

Join self with row vectors arg (in vertical way).

The function combines self with the last row vector of self. That is, extendRow(arg) is same to insertRow(self.row+1, arg).

arg must be list, **Vector** or **Matrix**. The length (or **column**) of it should be same to the column of self.

1.1.1.14 extendColumn – extend column vectors

```
extendColumn(self, arg: list/Vector/Matrix) → (None)
```

Join self with column vectors arg (in horizontal way).

The function combines self with the last column vector of self. That is,

`extendColumn(arg)` is same to `insertColumn(self.column+1, arg)`.

`arg` must be list, **Vector** or **Matrix**. The length (or **row**) of it should be same to the row of `self`.

1.1.1.15 deleteRow – delete row vector

`deleteRow(self, i: integer) → (None)`

Delete *i*-th row vector.

1.1.1.16 deleteColumn – delete column vector

`deleteColumn(self, j: integer) → (None)`

Delete *j*-th column vector.

1.1.1.17 transpose – transpose matrix

`transpose(self) → Matrix`

Return the transpose of `self`.

1.1.1.18 getBlock – block matrix

`getBlock(self, i: integer, j: integer, row: integer, column: integer=None) → Matrix`

Return the `row`×`column` block matrix from the (*i*, *j*)-element.

If `column` is omitted, `column` is considered as same value to `row`.

1.1.1.19 subMatrix – submatrix

`subMatrix(self, I: integer, J: integer=None) → Matrix`

`subMatrix(self, I: list, J: list=None) → Matrix`

The function has a twofold significance.

- I and J are integer:
Return submatrix deleted I-th row and J-th column.
- I and J are list:
Return the submatrix composed of elements from self assigned by rows I and columns J, respectively.

If J is omitted, J is considered as same value to I.

Examples

```
>>> A = matrix.Matrix(2, 3, [1,2,3]+[4,5,6])
>>> A
[1, 2, 3]+[4, 5, 6]
>>> A.map(complex)
[(1+0j), (2+0j), (3+0j)]+[(4+0j), (5+0j), (6+0j)]
>>> A.reduce(max)
6
>>> A.swapRow(1, 2)
>>> A
[4, 5, 6]+[1, 2, 3]
>>> A.extendColumn([-2, -1])
>>> A
[4, 5, 6, -2]+[1, 2, 3, -1]
>>> B = matrix.Matrix(3, 3, [1,2,3]+[4,5,6]+[7,8,9])
>>> B.subMatrix(2, 3)
[1, 2]+[7, 8]
>>> B.subMatrix([2, 3], [1, 2])
[4, 5]+[7, 8]
```

1.1.2 SquareMatrix – square matrices

Initialize (Constructor)

```
SquareMatrix(row: integer, column: integer=0, compo: compo=0,  
coeff_ring: CommutativeRing=0)  
→ SquareMatrix
```

Create new square matrices object.

SquareMatrix is subclass of **Matrix**. †This constructor automatically changes the class to one of the following class: **RingMatrix**, **RingSquareMatrix**, **FieldMatrix**, **FieldSquareMatrix**.

Your input determines the class automatically by examining the matrix size and the coefficient ring. row and column must be integer, and coeff_ring must be an instance of **Ring**. Refer to **compo** for information about compo. If you abbreviate compo, it will be deemed to all zero list.

The list of expected inputs and outputs is as following:

- Matrix(row, compo, coeff_ring)
→ the row square matrix whose elements are compo and coefficient ring is coeff_ring
- Matrix(row, compo)
→ the row square matrix whose elements are compo (coefficient ring is automatically determined)
- Matrix(row, coeff_ring)
→ the row square matrix whose coefficient ring is coeff_ring (All elements are 0 in coeff_ring.)
- Matrix(row)
→ the row square matrix (The coefficient ring is Integer. All elements are 0.)

†We can initialize as **Matrix**, but column must be same to row in the case.

Methods

1.1.2.1 isUpperTriangularMatrix – check upper triangular

`isUpperTriangularMatrix(self)` → *True/False*

Check whether `self` is upper triangular matrix or not.

1.1.2.2 isLowerTriangularMatrix – check lower triangular

`isLowerTriangularMatrix(self)` → *True/False*

Check whether `self` is lower triangular matrix or not.

1.1.2.3 isDiagonalMatrix – check diagonal matrix

`isDiagonalMatrix(self)` → *True/False*

Check whether `self` is diagonal matrix or not.

1.1.2.4 isScalarMatrix – check scalar matrix

`isScalarMatrix(self)` → *True/False*

Check whether `self` is scalar matrix or not.

1.1.2.5 isSymmetricMatrix – check symmetric matrix

`isSymmetricMatrix(self)` → *True/False*

Check whether `self` is symmetric matrix or not.

Examples

```
>>> A = matrix.SquareMatrix(3, [1,2,3]+[0,5,6]+[0,0,9])
>>> A.isUpperTriangularMatrix()
```

```
True
>>> B = matrix.SquareMatrix(3, [1,0,0]+[0,-2,0]+[0,0,7])
>>> B.isDiagonalMatrix()
True
```

1.1.3 RingMatrix – matrix whose elements belong ring

```
RingMatrix(row: integer, column: integer, compo: compo=0,  
coeff_ring: CommutativeRing=0)  
→ RingMatrix
```

Create matrix whose coefficient ring belongs ring.

RingMatrix is subclass of **Matrix**. See Matrix for getting information about the initialization.

Operations

operator	explanation
M+N	Return the sum of matrices M and N.
M-N	Return the difference of matrices M and N.
M*N	Return the product of M and N. N must be matrix, vector or scalar
M % d	Return M modulo d. d must be nonzero integer.
-M	Return the matrix whose coefficients have inverted signs of M.
+M	Return the copy of M.

Examples

```
>>> A = matrix.Matrix(2, 3, [1,2,3]+[4,5,6])  
>>> B = matrix.Matrix(2, 3, [7,8,9]+[0,-1,-2])  
>>> A + B  
[8, 10, 12]+[4, 4, 4]  
>>> A - B  
[-6, -6, -6]+[4, 6, 8]  
>>> A * B.transpose()  
[50, -8]+[122, -17]  
>>> -B * vector.Vector([1, -1, 0])  
Vector([1, -1])  
>>> 2 * A  
[2, 4, 6]+[8, 10, 12]  
>>> B % 3  
[1, 2, 0]+[0, 2, 1]
```


Methods

1.1.3.1 getCoefficientRing – get coefficient ring

`getCoefficientRing(self)` → *CommutativeRing*

Return the coefficient ring of self.

This method checks all elements of self and set `coeff_ring` to the valid coefficient ring.

1.1.3.2 toFieldMatrix – set field as coefficient ring

`toFieldMatrix(self)` → *(None)*

Change the class of the matrix to **FieldMatrix** or **FieldSquareMatrix**, where the coefficient ring will be the quotient field of the current domain.

1.1.3.3 toSubspace – regard as vector space

`toSubspace(self, isbasis: True/False=None)` → *(None)*

Change the class of the matrix to Subspace, where the coefficient ring will be the quotient field of the current domain.

1.1.3.4 hermiteNormalForm (HNF) – Hermite Normal Form

`hermiteNormalForm(self)` → *RingMatrix*

`HNF(self)` → *RingMatrix*

Return upper triangular Hermite normal form (HNF).

1.1.3.5 exthermiteNormalForm (extHNF) – extended Hermite Normal Form algorithm

`exthermiteNormalForm(self)` → *(RingSquareMatrix, RingMatrix)*

`extHNF(self)` → *(RingSquareMatrix, RingMatrix)*

Return Hermite normal form M and U satisfied $selfU = M$.

The function returns tuple (U, M) , where U is an instance of **RingSquareMatrix** and M is an instance of **RingMatrix**.

1.1.3.6 kernelAsModule – kernel as \mathbb{Z} -module

kernelAsModule(self) → RingMatrix

Return kernel as \mathbb{Z} -module.

The difference between the function and **kernel** is that each elements of the returned value are integer.

Examples

```
>>> A = matrix.Matrix(3, 4, [1,2,3,4,5,6,7,8,9,-1,-2,-3])
>>> print A.hermiteNormalForm()
0 36 29 28
0 0 1 0
0 0 0 1
>>> U, M = A.hermiteNormalForm()
>>> A * U == M
True
>>> B = matrix.Matrix(1, 2, [2, 1])
>>> print B.kernelAsModule()
1
-2
```

1.1.4 RingSquareMatrix – square matrix whose elements belong ring

```
RingSquareMatrix(row: integer, column: integer=0, compo:
compo=0, coeff_ring: CommutativeRing=0)
→ RingMatrix
```

Create square matrix whose coefficient ring belongs ring.

RingSquareMatrix is subclass of **RingMatrix** and **SquareMatrix**. See SquareMatrix for getting information about the initialization.

Operations

operator	explanation
M**c	Return the c-th power of matrices M.

Examples

```
>>> A = matrix.RingSquareMatrix(3, [1,2,3]+[4,5,6]+[7,8,9])
>>> A ** 2
[30L, 36L, 42L]+[66L, 81L, 96L]+[102L, 126L, 150L]
```

Methods

1.1.4.1 `getRing` – get matrix ring

`getRing(self)` → *MatrixRing*

Return the **MatrixRing** belonged to by `self`.

1.1.4.2 `isOrthogonalMatrix` – check orthogonal matrix

`isOrthogonalMatrix(self)` → *True/False*

Check whether `self` is orthogonal matrix or not.

1.1.4.3 `isAlternatingMatrix` (`isAntiSymmetricMatrix`, `isSkewSymmetricMatrix`) – check alternating matrix

`isAlternatingMatrix(self)` → *True/False*

Check whether `self` is alternating matrix or not.

1.1.4.4 `isSingular` – check singular matrix

`isSingular(self)` → *True/False*

Check whether `self` is singular matrix or not.

The function determines whether determinant of `self` is 0. Note that the non-singular matrix does not automatically mean invertible matrix; the nature that the matrix is invertible depends on its coefficient ring.

1.1.4.5 `trace` – trace

`trace(self)` → *RingElement*

Return the trace of `self`.

1.1.4.6 determinant – determinant

determinant(self) → *RingElement*

Return the determinant of self.

1.1.4.7 cofactor – cofactor

cofactor(self, i: *integer*, j: *integer*) → *RingElement*

Return the (i, j)-cofactor.

1.1.4.8 commutator – commutator

commutator(self, N: *RingSquareMatrix element*) → *RingSquareMatrix*

Return the commutator for self and N.

The commutator for M and N, which is denoted as $[M, N]$, is defined as $[M, N] = MN - NM$.

1.1.4.9 characteristicMatrix – characteristic matrix

characteristicMatrix(self) → *RingSquareMatrix*

Return the characteristic matrix of self.

1.1.4.10 adjugateMatrix – adjugate matrix

adjugateMatrix(self) → *RingSquareMatrix*

Return the adjugate matrix of self.

The adjugate matrix for M is the matrix N such that $MN = NM = (\det M)E$, where E is the identity matrix.

1.1.4.11 cofactorMatrix (cofactors) – cofactor matrix

`cofactorMatrix(self)` → *RingSquareMatrix*

`cofactors(self)` → *RingSquareMatrix*

Return the cofactor matrix of self.

The cofactor matrix for M is the matrix whose (i, j) element is (i, j) -cofactor of M. The cofactor matrix is same to transpose of the adjugate matrix.

1.1.4.12 smithNormalForm (SNF, elementary_divisor) – Smith Normal Form (SNF)

`smithNormalForm(self)` → *RingSquareMatrix*

`SNF(self)` → *RingSquareMatrix*

`elementary_divisor(self)` → *RingSquareMatrix*

Return the list of diagonal elements of the Smith Normal Form (SNF) for self.

The function assumes that self is non-singular.

1.1.4.13 extsmithNormalForm (extSNF) – Smith Normal Form (SNF)

`extsmithNormalForm(self)` → (*RingSquareMatrix, RingSquareMatrix, RingSquareMatrix*)

`extSNF(self)` → *RingSquareMatrix, RingSquareMatrix, RingSquareMatrix*

Return the Smith normal form M for self and U,V satisfied $UselfV = M$.

Examples

```
>>> A = matrix.RingSquareMatrix(3, [3,-5,8]+[-9,2,7]+[6,1,-4])
>>> A.trace()
1L
>>> A.determinant()
-243L
>>> B = matrix.RingSquareMatrix(3, [87,38,80]+[13,6,12]+[65,28,60])
>>> U, V, M = B.extsmithNormalForm()
>>> U * B * V == M
True
```

```
>>> print M
4 0 0
0 2 0
0 0 1
>>> B.smithNormalForm()
[4L, 2L, 1L]
```

1.1.5 FieldMatrix – matrix whose elements belong field

```
FieldMatrix(row: integer, column: integer, compo: compo=0,  
coeff_ring: CommutativeRing=0)  
→ RingMatrix
```

Create matrix whose coefficient ring belongs field.

FieldMatrix is subclass of **RingMatrix**. See **Matrix** for getting information about the initialization.

Operations

operator	explanation
M/d	Return the division of M by d.d must be scalar.
M//d	Return the division of M by d.d must be scalar.

Examples

```
>>> A = matrix.FieldMatrix(3, 3, [1,2,3,4,5,6,7,8,9])  
>>> A / 210  
1/210 1/105 1/70  
2/105 1/42 1/35  
1/30 4/105 3/70
```


Methods

1.1.5.1 kernel – kernel

kernel(self) → *FieldMatrix*

Return the kernel of **self**.

The output is the matrix whose column vectors form basis of the kernel. The function returns None if the kernel do not exist.

1.1.5.2 image – image

image(self) → *FieldMatrix*

Return the image of **self**.

The output is the matrix whose column vectors form basis of the image. The function returns None if the kernel do not exist.

1.1.5.3 rank – rank

rank(self) → *integer*

Return the rank of **self**.

1.1.5.4 inverseImage – inverse image: base solution of linear system

inverseImage(self, v: *Vector/RingMatrix*) → *RingMatrix*

Return an inverse image of **v** by **self**.

The function returns one solution of the linear equation $\mathit{self}X = v$.

1.1.5.5 solve – solve linear system

solve(self, B: *Vector/RingMatrix*) → (*RingMatrix*, *RingMatrix*)

Solve $\mathit{self}X = B$.

The function returns a particular solution **sol** and the kernel of **self** as a

matrix. If you only have to obtain the particular solution, use `inverseImage`.

1.1.5.6 `columnEchelonForm` – column echelon form

`columnEchelonForm(self)` → *RingMatrix*

Return the column reduced echelon form.

Examples

```
>>> A = matrix.FieldMatrix(2, 3, [1,2,3]+[4,5,6])
>>> print A.kernel
1/1
-2/1
1
>>> print A.image()
1 2
4 5
>>> C = matrix.FieldMatrix(4, 3, [1,2,3]+[4,5,6]+[7,8,9]+[-1,-2,-3])
>>> D = matrix.FieldMatrix(4, 2, [1,0]+[7,6]+[13,12]+[-1,0])
>>> print C.inverseImage(D)
3/1 4/1
-1/1 -2/1
0/1 0/1
>>> sol, ker = C.solve(D)
>>> C * (sol + ker[0]) == D
True
>>> AA = matrix.FieldMatrix(3, 3, [1,2,3]+[4,5,6]+[7,8,9])
>>> print AA.columnEchelonForm()
0/1 2/1 -1/1
0/1 1/1 0/1
0/1 0/1 1/1
```

1.1.6 FieldSquareMatrix – square matrix whose elements belong field

```
FieldSquareMatrix(row: integer, column: integer=0, compo:
compo=0, coeff_ring: CommutativeRing=0)
→ FieldSquareMatrix
```

Create square matrix whose coefficient ring belongs field.

FieldSquareMatrix is subclass of **FieldMatrix** and **SquareMatrix**.

†The function **RingSquareMatrix**determinant is overridden and use different algorithm from one used in **RingSquareMatrix**determinant;the function calls **FieldSquareMatrix**triangulate. See **SquareMatrix** for getting information about the initialization.

Methods

1.1.6.1 triangulate - triangulate by elementary row operation

triangulate(self) → *FieldSquareMatrix*

Return an upper triangulated matrix obtained by elementary row operations.

1.1.6.2 inverse - inverse matrix

inverse(self v: *Vector/RingMatrix=None*) → *FieldSquareMatrix*

Return the inverse of self. If v is given, then return $self^{-1}v$.

‡If the matrix is not invertible, then raise **NoInverse**.

1.1.6.3 hessenbergForm - Hessenberg form

hessenbergForm(self) → *FieldSquareMatrix*

Return the Hessenberg form of self.

1.1.6.4 LUdecomposition - LU decomposition

LUdecomposition(self) → (*FieldSquareMatrix*, *FieldSquareMatrix*)

Return the lower triangular matrix L and the upper triangular matrix U such that $self == LU$.

1.1.7 †MatrixRing – ring of matrices

```
MatrixRing(size: integer, scalars: CommutativeRing)  
→ MatrixRing
```

Create a ring of matrices with given size and coefficient ring scalars.

MatrixRing is subclass of **Ring**.

Methods

1.1.7.1 unitMatrix - unit matrix

`unitMatrix(self)` → *RingSquareMatrix*

Return the unit matrix.

1.1.7.2 zeroMatrix - zero matrix

`zeroMatrix(self)` → *RingSquareMatrix*

Return the zero matrix.

Examples

```
>>> M = matrix.MatrixRing(3, rational.theIntegerRing)
>>> print M
M_3(Z)
>>> M.unitMatrix()
[1L, 0L, 0L]+[0L, 1L, 0L]+[0L, 0L, 1L]
>>> M.zero
[0L, 0L, 0L]+[0L, 0L, 0L]+[0L, 0L, 0L]
```

1.1.7.3 getInstance(class function) - get cached instance

`getInstance(cls, size: integer, scalars: CommutativeRing)`
→ *RingSquareMatrix*

Return an instance of MatrixRing of given size and ring of scalars.

The merit of using the method instead of the constructor is that the instances created by the method are cached and reused for efficiency.

Examples

```
>>> print MatrixRing.getInstance(3, rational.theIntegerRing)
M_3(Z)
```

1.1.8 Subspace – subspace of finite dimensional vector space

```
Subspace(row: integer, column: integer=0, compo: compo=0,  
coeff_ring: CommutativeRing=0, isbasis: True/False=None)  
→ Subspace
```

Create subspace of some finite dimensional vector space over a field.

Subspace is subclass of **FieldMatrix**.

See **Matrix** for getting information about the initialization. The subspace expresses the space generated by column vectors of `self`.

If `isbasis` is `True`, we assume that column vectors are linearly independent.

Attributes

isbasis The attribute indicates the linear independence of column vectors, i.e., if they form a basis of the space then `isbasis` should be `True`, otherwise `False`.

Methods

1.1.8.1 issubspace - check subspace of self

Subspace(self, other: *Subspace*) → *True/False*

Return True if the subspace instance is a subspace of the other, or False otherwise.

1.1.8.2 toBasis - select basis

toBasis(self) → (*None*)

Rewrite self so that its column vectors form a basis, and set True to its isbasis.

The function does nothing if isbasis is already True.

1.1.8.3 supplementBasis - to full rank

supplementBasis(self) → *Subspace*

Return full rank matrix by supplementing bases for self.

1.1.8.4 sumOfSubspaces - sum as subspace

sumOfSubspaces(self, other: *Subspace*) → *Subspace*

Return a matrix whose columns form a basis for sum of two subspaces.

1.1.8.5 intersectionOfSubspaces - intersection as subspace

intersectionOfSubspaces(self, other: *Subspace*) → *Subspace*

Return a matrix whose columns form a basis for intersection of two subspaces.

Examples

```
>>> A = matrix.Subspace(4, 3, [1,2,3]+[4,5,6]+[7,8,9]+[10,11,12])
>>> A.toBasis()
>>> print A
1 2
4 5
7 8
10 11
>>> B = matrix.Subspace(3, 2, [1,2]+[3,4]+[5,7])
>>> print B.supplementBasis()
1 2 0
3 4 0
5 7 1
>>> C = matrix.Subspace(4, 1, [1,2,3,4])
>>> D = matrix.Subspace(4, 2, [2,-4]+[4,-3]+[6,-2]+[8,-1])
>>> print C.intersectionOfSubspaces(D)
-2/1
-4/1
-6/1
-8/1
```

1.1.8.6 fromMatrix(class function) - create subspace

```
fromMatrix(cls, mat: FieldMatrix, isbasis: True/False=None)  
→ Subspace
```

Create a Subspace instance from a matrix instance mat, whose class can be any of subclasses of Matrix.

Please use this method if you want a Subspace instance for sure.

1.1.9 createMatrix[function] – create an instance

```
createMatrix(row: integer, column: integer=0, compo: compo=0,  
             coeff_ring: CommutativeRing=None)  
             → RingMatrix
```

Create an instance of **RingMatrix**, **RingSquareMatrix**, **FieldMatrix** or **FieldSquareMatrix**.

Your input determines the class automatically by examining the matrix size and the coefficient ring. See **Matrix** or **SquareMatrix** for getting information about the initialization.

1.1.10 identityMatrix(unitMatrix)[function] – unit matrix

```
identityMatrix(size: integer, coeff: CommutativeR-  
              ing/CommutativeRingElement=None)  
              → RingMatrix
```

```
unitMatrix(size: integer, coeff: CommutativeR-  
           ing/CommutativeRingElement=None)  
           → RingMatrix
```

Return size-dimensional unit matrix.

coeff enables us to create matrix not only in integer but in coefficient ring which is determined by coeff.

coeff must be an instance of **Ring** or a multiplicative unit (one).

1.1.11 zeroMatrix[function] – zero matrix

```
zeroMatrix(row: integer, column: 0=, coeff: CommutativeR-  
          ing/CommutativeRingElement=None)  
          → RingMatrix
```

Return $row \times column$ zero matrix.

coeff enables us to create matrix not only in integer but in coefficient ring which is determined by coeff.

coeff must be an instance of **Ring** or a additive unit (zero). If column is abbreviated, column is set same to row.

Examples

```
>>> M = matrix.createMatrix(3, [1,2,3]+[4,5,6]+[7,8,9])
>>> print M
1 2 3
4 5 6
7 8 9
>>> 0 = matrix.zeroMatrix(2, 3, imaginary.ComplexField())
>>> print 0
0 + 0j 0 + 0j 0 + 0j
0 + 0j 0 + 0j 0 + 0j
```

Bibliography